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Testing Gauge Theories in Electron-Electron Scattering

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Abstract

The magnitudes of the correction terms to Møller scattering in the Weinberg–Salam Model and Georgi–Glashow model are compared. The possible consequences of the variations of the free parameters M_{E^+}/M_W, E/M_ϕ and E/M_Z with these correction terms are carefully discussed.

1. *Introduction*

As emphasized in a recent preprint by Dass, (1973), electron-electron scattering provides a good testing ground for a number of unified gauge models of weak and electromagnetic interactions (Abers and Lee, 1973; Georgi and Glashow, 1972; Weinberg, 1971; Salam, 1968; Sehgal, 1973; Murtaza, 1974; Beg and Sirlin, 1974; Mahanthappa, 1973; MacDowel, 1974). In particular both Weinberg-Salam model and Georgi-Glashow model contain terms which describe the $e^-e^- \to e^-e^-$ process. In this paper we want to compare the magnitudes of the corrections to the normal electromagnetic process evaluated in these two models. We will also study in some details the variation of these correction terms with the free parameters M_{E^+}/M_W, E/M_ϕ and E/M_Z where M_{E^+} is the mass of the heavy lepton, M_ϕ is the mass of the physical scalar meson; M_Z, M_W are the masses of the neutral and charged vector meson; E is the lab energy.

2. *Calculation of the Correction Terms*

In the Weinberg–Salam model (Weinberg, 1971; Salam, 1968), the general form of the Lagrangian which describes the process $e^-e^- \to e^-e^-$ is given by

$$L = e_0 A_\mu \bar{e}\gamma_\mu e + f\bar{e}\gamma_\mu(g_v + g_A\gamma_5)eZ_\mu \tag{2.1}$$

The effect of the physical scalar φ is negligibly small and is usually left out. In our previous paper (Ndili, Okeke, and Chukwumah, 1975), we showed using the above Lagrangian that the spin averaged differential cross section can be written in the form

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_\gamma [1 + \delta_{\gamma Z} + \delta_Z] \tag{2.2}$$

where $(d\sigma/d\Omega)_\gamma$ is the high energy limit of Møller cross section, $\delta\gamma z$ and δz are correction terms to the cross section. We showed that δz is negligible and that in the limit $E^2/M_Z^2 \ll 1$, $\delta\gamma z$ is approximately given by

$$\delta\gamma z = \frac{f^2}{e_0^2} \frac{E^2}{M_Z^2} (1 - z^2) \frac{[8k_1 + (1 + z)^3 k_2]}{(3 + z^2)^2} + (z \to -z) \tag{2.3}$$

where $z = \cos\theta$, θ being the scattering angle.

Consider next the Georgi-Glashow model (Georgi and Glashow, 1972) which is based on the group $SO(3)$ and requires the introduction of heavy leptons E^+ and E^0, M^+ and M^0, apart from the usual leptons e, νe, μ and ν_μ. The electron multiplet classifications are:

$$L = \begin{pmatrix} E_L^+ \\ X_L \\ e_L \end{pmatrix} \qquad R = \begin{pmatrix} E_R^+ \\ E_R^0 \\ e_R \end{pmatrix}$$

where $X_L = \nu_L \sin\beta + E_L^0 \cos\beta$ and the electron singlet $Y_L = E_L^0 \sin\beta - \nu_L \cos\beta$. Corresponding multiplets occur for the muon. In the explicit form, we obtain the following Lagrangian for the purely leptonic processes.

$$\begin{aligned}
L_{G\cdot G} = {} & -\tfrac{1}{4}F_{\mu\nu}^i F^{i\mu\nu} \\
& + \bar{e} i\gamma_\mu \partial_\mu e + \bar{E}^+ i\gamma_\mu \partial_\mu E^+ + \bar{E}_i^0 \gamma_\mu \partial_\mu E^0 \\
& - \bar{E}^0 \frac{(1+\gamma_5)}{2} i\gamma_\mu \partial_\mu E^0 \sin^2\beta + (\bar{\nu}\sin\beta + \bar{E}^0\cos\beta)\frac{(1+\gamma_5)}{2} i\gamma_\mu \partial_\mu \sin\beta \\
& + \bar{\nu}\sin\beta \frac{(1+\gamma_5)}{2} i\gamma_\mu \partial_\mu E^0 \cos\beta + e_0 A_\mu [\bar{E}^+ \gamma_\mu E^+ - \bar{e}\gamma_\mu e] \\
& + \frac{e_0 W^+}{2} [\bar{E} + \gamma_\mu (1-\gamma_5)\nu \sin\beta + \bar{E} + \gamma_\mu (1-\gamma_5) E^0 \cos\beta \\
& + \bar{\nu}\sin\beta\gamma_\mu(1-\gamma_5)e - E^0\cos\beta\gamma_\mu(1-\gamma_5)e + \bar{E} + \gamma\mu(1+\gamma_5)E^0 \\
& - \bar{E}^0\gamma_\mu(1+\gamma_5)e] \\
& + \frac{e_0 W^-}{2} [\bar{\nu}\sin\beta\gamma_\mu(1-\gamma_5)E^+ + \bar{E}^0\cos\beta\gamma_\mu(1-\gamma_5)E^+ + \bar{e}\gamma_\mu(1-\gamma_5)\sin\beta \\
& - \bar{e}\gamma_\mu(1-\gamma_5)E^0\cos\beta + \bar{E}^0\gamma_\mu(1+\gamma_5)E^+ - \bar{e}\gamma_\mu(1+\gamma_5)E^0] \\
& + \tfrac{1}{4}(\partial\varphi)^2 - \tfrac{1}{4}g^2(\varphi+\eta)W^{+2} - \tfrac{1}{4}g^2(\varphi+\eta)W^{-2}
\end{aligned}$$

$$-\frac{\mu^2}{\sqrt{2}}(\varphi+\eta)^2-\frac{\lambda}{2}(\varphi+\eta)^4$$

$$-\bar{E}+E+\left[m_0-\frac{G_1}{\sqrt{2}}(\varphi+\eta)\right]-\bar{e}\,e\left[m_0+\frac{G_1}{\sqrt{2}}(\varphi+\eta)\right]$$

$$-\bar{E}^0E^0\left[\frac{G_2}{\sqrt{2}}(\varphi+\eta)\sin\beta+m_0\cos\beta\right]-\frac{G_2}{\sqrt{2}}(\varphi+\eta)\bar{E}^0\nu\cos\beta$$

$$+\tfrac{1}{2}\sin\beta[\bar{\nu}(1+\gamma_5)E^0+\bar{E}^0(1-\gamma_5)\nu]$$

+ (a corresponding expression for the muon and its set of heavy leptons and the muon neutrino) (2.4)

Where

$$F^i_{\mu\nu}=\partial_\mu A^i_\nu-\partial_\nu A^i_\mu+g\epsilon^{ijk}A^j_\mu A^k_\nu \tag{2.5}$$

From the Lagrangian it is easy to see that

$$G_1=e_0\frac{(M_{E^+}-m_e)}{2M_W} \tag{2.6}$$

where e_0 is the electromagnetic coupling constant. The relevant part of the Lagrangian for the process $e^-e^-\to e^-e^-$ is given by

$$L=-e_0A_\mu\bar{e}\gamma_\mu e-e_0\frac{(M_{E^+}-m_e)}{2M_W}\varphi\bar{e}\,e \tag{2.7}$$

Using this Lagrangian and calculating in the relativistic limit $E\gg m$, with the same kinematics as used in our previous paper (Ndili, Okeke, Chukwumah, 1975), the Georgi-Glashow spin-averaged differential cross section for the process can be shown to be

$$\frac{d\sigma}{d\Omega}=\frac{\alpha^2(3+z^2)^2}{4E^2(1-z^2)^2}$$

$$+\frac{\alpha^2}{64}\frac{M^4_{E^+}}{M^4_W}E^2\left\{\frac{(1+z)^2}{[2E^2(1+z)+M^2_\varphi]^2}+\frac{(1-z)^2}{[2E^2(1-z)+M^2_\varphi]^2}\right.$$

$$\left.+\frac{(1-z)^2}{[2E^2(1+z)+M^2_\varphi]\,[2E^2(1-z)+M^2_\varphi]}\right\}$$

$$+\frac{\alpha^2}{16}\frac{M^2_{E^+}}{M^2_W}\frac{(1-z)^2}{(1+z)[2E^2(1-z)+M^2_\varphi]^2}+(z\to -z) \tag{2.8}$$

This cross section has been previously calculated by Dicus (1973). We can write this cross section in the form

$$\frac{d\sigma}{d\Omega}=\left(\frac{d\sigma}{d\Omega}\right)_\gamma[1+\delta_{\gamma\varphi}+\delta_\varphi] \tag{2.9}$$

where $\delta_{\gamma\varphi}$ and δ_φ are the φ correction terms.

After some algebra we get that

$$\delta_{\gamma\varphi} = \frac{M_E^2 + E^2}{4M_W^2}\left\{\frac{(1-z)^4(1+z)}{(3+z^2)^2[2E^2(1-z)+M_\varphi^2]} + (z \to -z)\right\} \tag{2.10a}$$

$$\delta_\varphi = \frac{M_{E^+}^4}{16M_w^4}\frac{(1-z^2)^2}{(3+z^2)^2}\left\{\frac{(1+z)^2}{[2E^2(1+z)+M_\varphi^2]^2} + \frac{(1-z)^2}{[2E^2(1-z)+M_\varphi^2]^2}\right.$$
$$\left. + \frac{(1-z)^2}{[2E^2(1+z)+M_\varphi^2]\,[2E^2(1-z)+M_\varphi^2]}\right\} + (z \to -z) \tag{2.10b}$$

3. *Discussion and Numerical Checks*

We now wish to compare the corrections to the Mϕller scattering predicted by the two models. We note immediately from equations (2.3) and (2.8) that in the limit $z \to 1$ all the correction terms are zero.
That is

$$\delta_{\gamma z} = \delta_{\gamma\varphi} = \delta_\varphi = 0 \tag{3.1}$$

Next, introducing the following free parameters

$$R = \frac{E^2}{M_\varphi^2}, \qquad T = \frac{M_{E^+}^2}{M_W^2}, \qquad K = \frac{E^2}{M_Z^2} \tag{3.2}$$

We can rewrite the correction terms as follows:
(in the limit $z \to 0$)

$$\delta_{\gamma z} \simeq \tfrac{8}{9}k \tag{3.3}$$

$$\delta_{\gamma\varphi} = \frac{T}{18(2+1/R)} \tag{3.4}$$

$$\delta_\varphi = \frac{T^2}{24(2+1/R)^2} \tag{3.5}$$

4. *Constraint on R and T*

First we consider the possible limit on these free parameters. The mass M_φ is not known but it can be as large as the lab energy E, as pointed out by Dass (1973). Thus the parameter $R = E^2/M_\varphi^2$ remains a free parameter. On the other hand, the specific value of M_{E^+} and M_W have not yet been fixed by experiments (Bjorken and Smith, 1973; Barish, et al., 1973; Golovkin et al., 1972; Albright, 1972). One may assume (Georgi and Glashow, 1972) 10*Gev*

$< M_W \leqslant 53$ *Gev* and at least $M_{E^+} > 1$ *Gev*. From the expected decay modes of M_{E^+}, one may probably say that $M_W > M_{E^+}$, since the process $E^+ \rightarrow W +$ lepton has not been observed. We therefore see that

$$T = \frac{M_{E^+}}{M_W} \tag{4.1}$$

is also a free parameter.

Now consider equations (3.3), (3.4) and (3.5). We find that as $R \rightarrow 0$, $\delta_{\gamma\varphi} = \delta_\varphi = 0$. That is, if the lab energy is very small compared with the mass of the physical scalar, the correction terms will be negligibly small. If $R \triangleq 1$, that is, if we choose the lab energy to be of the same order of magnitude as the mass of the physical scalar, then

$$\delta_{\gamma\varphi} = \frac{T}{54} \text{ and } \delta_\varphi = \frac{T^2}{216} \tag{4.2}$$

On the other hand if we increase E such that

$$E \gg M_\varphi, \delta_{\gamma\varphi} = \frac{T}{36} \tag{4.3}$$

and

$$\delta_\varphi = \frac{T^2}{96} \tag{4.4}$$

From the above equations (4.2) (4.3) and (4.4) we conclude that if R is fractional then the correction term seriously depends on R and diminishes as R becomes smaller and smaller; but for $R > 1$ the variation of $\delta_{\gamma\varphi}$ and δ_φ with R is fairly constant, and is completely independent of R as R becomes very large. Therefore in the range $1 < R < \infty$ the correction terms are approximately given by (4.4) and (4.3). That is, $\delta_{\gamma\varphi}$ depends linearly on T, while δ_φ depends quadratically on T.

5. *Variation of Correction Terms with T in the Range $1 < R < \infty$*

Since we are working under the constraint $M_{E^+} \leqslant M_W$, we may consider assigning the following numerical values to T

$$T = 1, 0.5, 0.2, 0.1, 0.05, 0.01 \tag{5.1}$$

The resulting correction terms are as shown in Table 1.

Compared with the Weinberg model, where the magnitude of the correction term depends largely on E, we find that in the Georgi-Glashow model, the correction terms depend largely on M_{E^+}. To achieve the 4% correction term discussed in our previous paper (Ndili, Okeke, Chukwumah, 1975), one would need in the Georgi-Glashow model to set $M_{E^+} > 50$ *Gev*.

TABLE 1. Variation of Correction Terms with T

Value of T	$\delta_{\gamma\varphi}(\%)$	$\delta_{\varphi}(\%)$	Total correction term (%)
1	2.77	1.04	3.81
0.5	1.38	0.26	1.64
0.2	0.55	0.04	0.59
0.1	0.28	0.01	0.29
0.05	0.14	0.002	0.14
0.01	0.02	0.0001	0.02

6. *Conclusion*

The magnitude of the correction term to the normal electromagnetic process evaluated in the Georgi-Glashow model increases as M_{E^+} becomes large, but there is a limit to which $R(=E^2/M_\varphi)$ can be increased without achieving any increase in the correction term. This feature may be compared with that in the Weinberg model where a large correction term is obtained as E becomes large compared with M_Z. Also, as expected, these results are enhanced if one is working off the forward direction.

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